

$$\hat{x} = 1 + 2\varepsilon_1 + 3\varepsilon_2 - 4\varepsilon_3$$

$\varepsilon_i \in [0, 1]$

$$\varepsilon_1 = 0, \varepsilon_2 = 1, \varepsilon_3 = 0$$

$$x = 4$$

$$\varepsilon_1 = -1, \varepsilon_2 = 0, \varepsilon_3 = 0$$

$$x = -1$$

$$\hat{\varepsilon}_1 = -0.223, \varepsilon_2 = 0.45, \varepsilon_3 = -0.668$$

$$\hat{x} = 1 - 2\varepsilon_1 + \varepsilon_2$$

$$\hat{y} = 0 + \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_2 + \varepsilon_1$$

$$\varepsilon_1 = 1, \varepsilon_2 = 1$$

$$(x, y) = (0, 4)$$

$$\varepsilon_1 = -1, \varepsilon_2 = -1$$

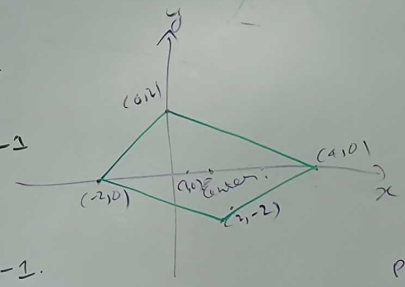
$$(x, y) = (2, -2)$$

$$\varepsilon_1 = 1, \varepsilon_2 = -1$$

$$(x, y) = (-2, 0)$$

$$\varepsilon_1 = -1, \varepsilon_2 = 1$$

$$(x, y) = (4, 0)$$



$$p = (0, 2)$$

$$2x + y = (2, 0) - (0, 2)$$

$$= (2, -2)$$

$$\hat{x} = 1 + \varepsilon_1 + \varepsilon_2$$

$$\hat{y} = 2 + 2\varepsilon_1 + 2\varepsilon_2$$

$$\varepsilon_1 = 1, \varepsilon_2 = 1$$

$$(x, y) = (3, 6)$$

$$\varepsilon_1 = -1, \varepsilon_2 = -1$$

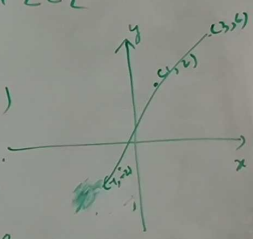
$$(x, y) = (1, -2)$$

$$\varepsilon_1 = 1, \varepsilon_2 = -1$$

$$(x, y) = (1, 2)$$

$$\varepsilon_1 = -1, \varepsilon_2 = 1$$

$$(x, y) = (1, 2)$$



$$\hat{x} = 1 + 2\varepsilon_1 + 3\varepsilon_2 - 4\varepsilon_3$$

$$\varepsilon_1 = 0, \varepsilon_2 = 1, \varepsilon_3 = 0$$

$$\hat{x} = 4$$

$$\varepsilon_1 = -1, \varepsilon_2 = 0, \varepsilon_3 = 0$$

$$x = -1$$

$$\varepsilon_1 = -0.223, \varepsilon_2 = 0.45, \varepsilon_3 = -0.668$$

$$\hat{x} = 1 - 2\varepsilon_1 + \varepsilon_2$$

$$\hat{y} = 0 + \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_2 + \varepsilon_1$$

$$\varepsilon_i \in [-1, 1]$$

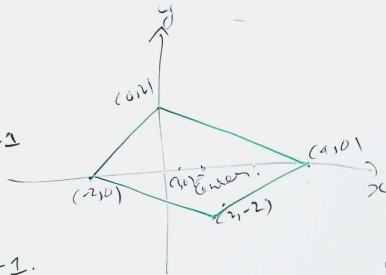
$$\begin{aligned} \varepsilon_1 = 1, \varepsilon_2 = 1 \\ (x, y) = (0, 4) \\ \varepsilon_1 = -1, \varepsilon_2 = -1 \\ (x, y) = (2, -2) \end{aligned}$$

$$\varepsilon_1 = 1, \varepsilon_2 = -1$$

$$(x, y) = (-2, 0)$$

$$\varepsilon_1 = -1, \varepsilon_2 = 1$$

$$(x, y) = (2, 0)$$



$$P = (0, 2)$$

$$\begin{aligned} 2 \times C &= (2, 0) - (0, 2) \\ &= (2, -2) \end{aligned}$$

$$\begin{aligned} \hat{x} &= 1 + \varepsilon_1 + \varepsilon_2 \\ \hat{y} &= 2 + 2\varepsilon_1 + 2\varepsilon_2 \end{aligned}$$

$$\begin{aligned} \varepsilon_1 = 1, \varepsilon_2 = 1 \\ (x, y) = (2, 6) \end{aligned}$$

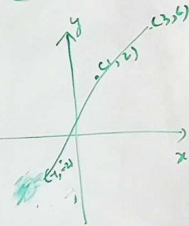
$$\begin{aligned} \varepsilon_1 = -1, \varepsilon_2 = -1 \\ (x, y) = (0, -2) \end{aligned}$$

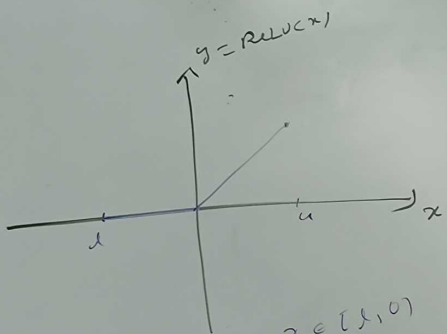
$$\varepsilon_1 = 1, \varepsilon_2 = -1$$

$$(x, y) = (2, 2)$$

$$\varepsilon_1 = -1, \varepsilon_2 = 1$$

$$(x, y) = (0, 2)$$





$$z \in [l, 0)$$

$$y = 0$$

$$\hat{x} = \frac{l}{2} + \frac{l}{2} \epsilon$$

$$y = 0$$

$$x = [0, u)$$

$$y = x$$

$$[a, b] \Rightarrow \left(\frac{a+b}{2}\right) + \frac{b-a}{2} \epsilon$$

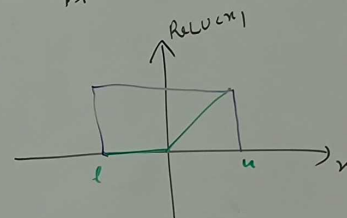
$$b \quad \epsilon = 1$$

$$a \quad \epsilon = -1$$

$$[a, b]$$

$$\hat{x} = a_0 + \sum_{i=1}^n \alpha_i \epsilon_i$$

$$\left[a_0 - \sum_{i=1}^n |\alpha_i|, a_0 + \sum_{i=1}^n |\alpha_i| \right]$$

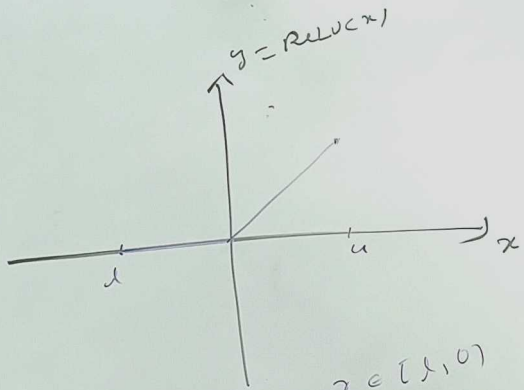


$$\hat{x} = \frac{4}{2} + \frac{4}{2} \epsilon$$

$$\hat{y} = \hat{x} = \frac{4}{2} + \frac{4}{2} \epsilon$$

$$\epsilon_1 = 1 \Rightarrow (x, y) = (4, 4)$$

$$\epsilon_1 = -1 \Rightarrow (x, y) = (0, 0)$$



$$z \in [l, 0)$$

$$y = 0.$$

$$\hat{x} = \frac{l}{2} + \frac{l}{2} \epsilon.$$

$$\hat{y} = 0$$

$$x \in [0, u)$$

$$y = x.$$

$$\hat{x} = \frac{u}{2} + \frac{u}{2} \epsilon.$$

$$\hat{y} = \hat{x} = \frac{u}{2} + \frac{u}{2} \epsilon.$$

$$[a, b] \Rightarrow \left(\frac{a+b}{2} \right) + \frac{(b-a)}{2} \epsilon$$

$$\epsilon_i = 1$$

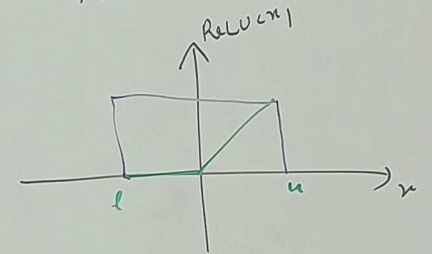
$$\epsilon_i = -1$$

$$a$$

$$[a, b]$$

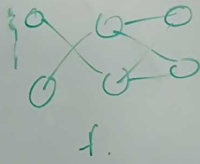
$$\hat{x} = d_0 + \sum_{i=1}^n \alpha_i \epsilon_i$$

$$\left[d_0 - \sum_{i=1}^n |\alpha_i|, d_0 + \sum_{i=1}^n |\alpha_i| \right]$$



$$\epsilon_i = 1 \Rightarrow (x, y) = (u, u)$$

$$\epsilon_i = -1 \Rightarrow (x, y) = (0, 0)$$



$$f(\Phi) \subseteq \Psi$$

$$f^{\#}(I)$$

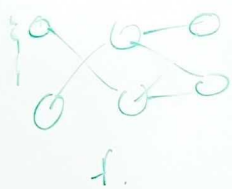
$$I \rightarrow Y(I) = x \Rightarrow f(x) \Rightarrow f(Y(I))$$

$$f^{\#}(I) \Rightarrow Y(f^{\#}(I)) \subseteq$$

$$I = -2 \leq x \leq 4.$$

$$x = \{ p \mid -2 \leq p \leq 4 \} = [-2, 4] = [-2, -1.999] \cup [-1.999, -1.999] \cup [-1.999, -1.998] \cup \dots \cup [3.01, 4]$$

$$f = \text{ReLU} = [\text{ReLU}(-2), \text{ReLU}(-1.999), \dots, \text{ReLU}(3.01)]$$



$$f(\Phi) \subseteq \Psi$$

$f^\#(I)$

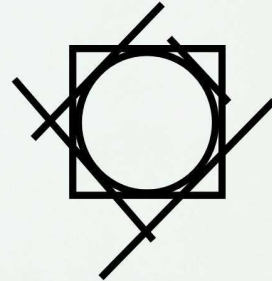
$$I \rightarrow Y(I) = x \Rightarrow f(x) \Rightarrow f(Y(I))$$

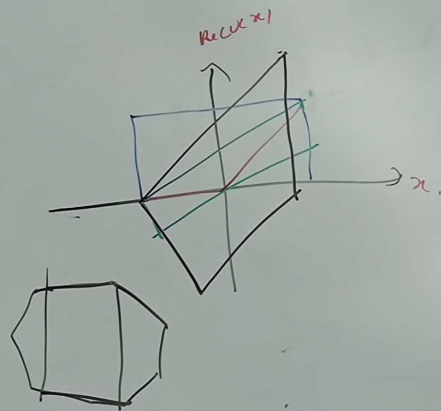
$$f^\#(I) \Rightarrow Y(f^\#(I)) \stackrel{=} {=} f(Y(I))$$

$$I = -2 \leq x \leq 4$$

$$x = \{ p \mid -2 \leq p \leq 4 \}, -2, -1.99, \dots, 4, 3.01, \dots$$

$$f = \text{ReLU} = \text{ReLU}(-2), \text{ReLU}(-1.99), \dots, \text{ReLU}(3.01)$$





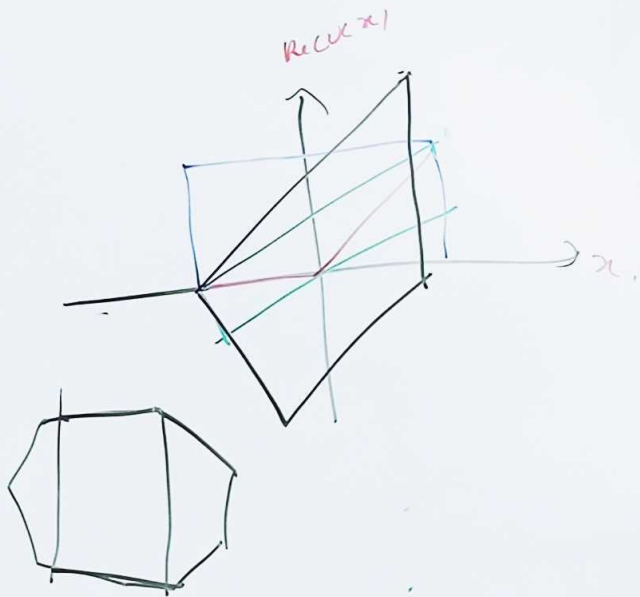
Whenever you are exact \Rightarrow Sound.
Optimal \Rightarrow Sound.

exact \Rightarrow Optimal.

Optimality \nRightarrow exactness.

Sound \nRightarrow Exact

Sound \nRightarrow optimal.



Whenever you are exact \Rightarrow Sound.
 Optimal \Rightarrow Sound.

exact \Rightarrow Optimal.
 optimality \nRightarrow exactness.
 sound \nRightarrow exact
 sound \nRightarrow optimal.

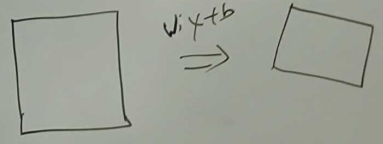


$$x^* \left(\alpha_0 + \sum_{i=1}^n \alpha_i \xi_i \right) \in \lambda \alpha_0 + \sum_{i=1}^n \lambda \alpha_i \xi_i$$

$$x = 1 - \varepsilon_1 + \varepsilon_2 \quad \varepsilon_1 = 1, \varepsilon_2 = 1$$

$$y = 1 + \varepsilon_1 - \varepsilon_2$$

$$x+y = (1+1) + (-1+1)\varepsilon_1 + (1-1)\varepsilon_2 = 2$$



$$x + \left(\alpha_0 + \sum_{i=1}^n \alpha_i \varepsilon_i \right) = \alpha_0 + \sum_{i=1}^n \alpha_i \varepsilon_i$$

(a, b)

$$x = 1 - \varepsilon_1 + \varepsilon_2 \quad \text{if } \varepsilon_1 = 1, \varepsilon_2 = 1 \quad \begin{matrix} 1 \\ 1 \end{matrix}$$

$$y = 1 + \varepsilon_1 - \varepsilon_2 \quad \begin{matrix} 1 \\ 1 \end{matrix}$$

$$x+y = (1+1) + (-1+1)\varepsilon_1 + (1-1)\varepsilon_2 = 2.$$

