$\hat{x} = 1 + 2\varepsilon_1 + 3\varepsilon_2 - 4\varepsilon_3$

$\varepsilon_i \in [-1, 1]$

$\varepsilon_1 = 0, \varepsilon_2 = 1, \varepsilon_3 = 0$

$x = 4$

$\varepsilon_1 = -1, \varepsilon_2 = 0, \varepsilon_3 = 0$

$x = -1.$

$\varepsilon_1 = -0.223, \varepsilon_2 = 0.45, \varepsilon_3 = -0.668.$

$\hat{x} = 1 - 2\varepsilon_1 + \varepsilon_2$

$\hat{y} = 0 + \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_3 + \varepsilon_4$

$\varepsilon_1 = 1, \varepsilon_2 = 1$

$(x,y) = (0, 2)$

$\varepsilon_1 = -1, \varepsilon_2 = -1$

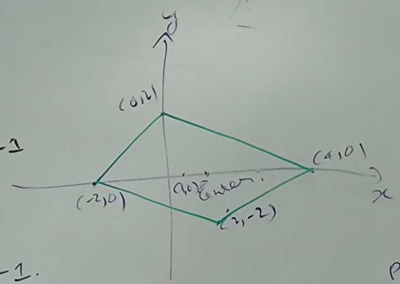
$(x,y) = (2,-2)$

$\varepsilon_1 = 1, \varepsilon_2 = -1.$

$(x,y) = (-2, 0)$

$\varepsilon_1 = -1, \varepsilon_2 = 1.$

$(x,y) = (4, 0)$



$p = (0,2)$

$2 \times c = (2,0) - (0,2)$

$= (2,-2)$

$\hat{x} = 1 + \varepsilon_1 + \varepsilon_2$

$\hat{y} = 2 + 2\varepsilon_1 + 2\varepsilon_2$

$\varepsilon_1 = 1, \varepsilon_2 = 1$

$(x,y) = (3, 6)$

$\varepsilon_1 = -1, \varepsilon_2 = -1.$

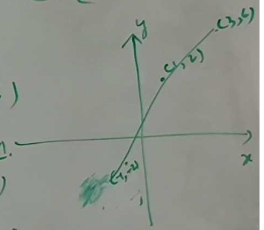
$(x,y) = (-1, -2)$

$\varepsilon_1 = 1, \varepsilon_2 = -1.$

$(x,y) = (1, 2)$

$\varepsilon_1 = -1, \varepsilon_2 = 1$

$(x,y) = (1,2)$



$\hat{x} = 1 + 2\varepsilon_1 + 3\varepsilon_2 - 4\varepsilon_3$

$\varepsilon_i \in [-1, 1]$

$\varepsilon_1 = 0, \varepsilon_2 = 1, \varepsilon_3 = 0$

$x = 4$

$\varepsilon_1 = -1, \varepsilon_2 = 0, \varepsilon_3 = 0$

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$\hat{x} = 1 - 2\varepsilon_1 + \varepsilon_2$

$\hat{y} = 0 + \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_3 + \varepsilon_4$

$\varepsilon_1 = 1, \varepsilon_2 = 1$

$(x,y) = (0, 2)$

$\varepsilon_1 = -1, \varepsilon_2 = -1$

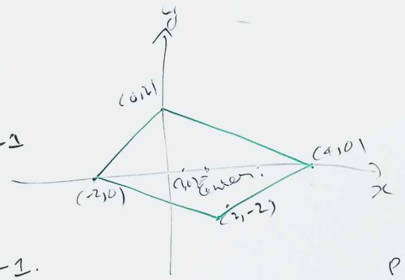
$(x,y) = (2, -2)$

$\varepsilon_1 = 1, \varepsilon_2 = -1.$

$(x,y) = (-2, 0)$

$\varepsilon_1 = -1, \varepsilon_2 = 1.$

$(x,y) = (4, 0)$



$p = (0,2)$

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$= (2,-2)$

$\hat{x} = 1 + \varepsilon_1 + \varepsilon_2$

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$\varepsilon_1 = 1, \varepsilon_2 = 1$

$(x,y) = (3, 6)$

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$(x,y) = (-1, -2)$

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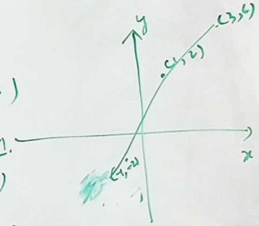
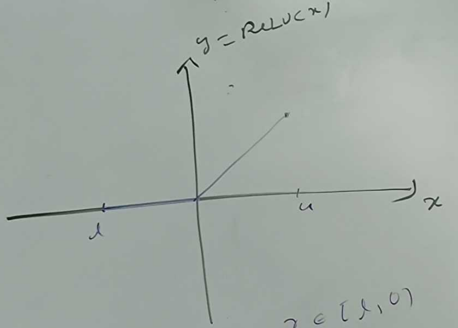
$(x,y) = (1, 2)$

$\varepsilon_1 = -1, \varepsilon_2 = 1$

$(x,y) = (1, 2)$





$[a, b] \Rightarrow \left(\frac{a+b}{2}\right) + \frac{(b-a)}{2}\varepsilon$

$\varepsilon_i = 1$ b

$\varepsilon = -1$ a

$[a, b]$

$x \in [l, 0)$

$y = 0.$

$\hat{x} = \frac{l}{2} + \frac{l}{2}\varepsilon.$

$\hat{y} = 0$

$x = [0, u)$

$y = x.$

$\hat{x} = \frac{u}{2} + \frac{u}{2}\varepsilon.$

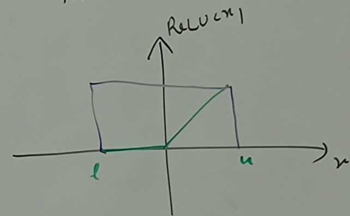
$\hat{y} = \hat{x} = \frac{u}{2} + \frac{u}{2}\varepsilon.$

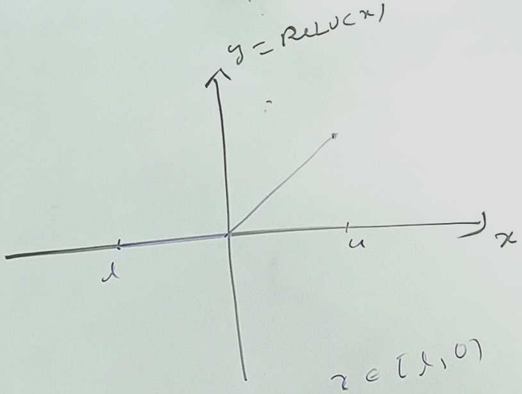
$\varepsilon_1 = 1. \Rightarrow (x, y) = (u, u)$

$\varepsilon = -1 \Rightarrow (x, y) = (0, 0)$

$\hat{x} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_i$

$[\alpha_0 - \sum_{i=1}^{n} |\alpha_i|, \alpha_0 + \sum_{i=1}^{n} |\alpha_i|]$





$x \in [l, 0)$

$y = 0.$

$\hat{x} = \frac{l}{2} + \frac{l}{2}\varepsilon.$

$\hat{y} = 0$

$x \in [0, u)$

$y = x.$

$[a, b) \Rightarrow \left(\frac{a+b}{2}\right) + \frac{(b-a)}{2}\varepsilon$

$\varepsilon_i = 1$

b

$\varepsilon = -1$

a

$[a, b)$

$\hat{x} = \frac{u}{2} + \frac{u}{2}\varepsilon.$

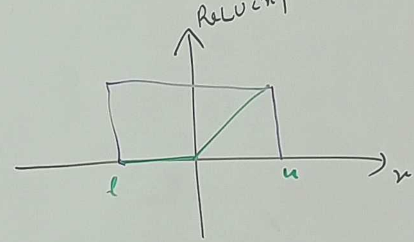
$\hat{y} = \hat{x} = \frac{u}{2} + \frac{u}{2}\varepsilon.$

$\varepsilon_1 = 1. \Rightarrow (x, y) = (u, u)$

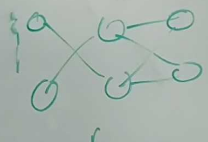
$\varepsilon = -1 \Rightarrow (x, y) = (0, 0)$

$\hat{x} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_i$

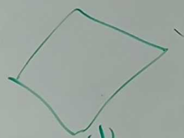
$[\alpha_0 - \sum_{i=1}^{n} |\alpha_i|, \alpha_0 + \sum_{i=1}^{n} |\alpha_i|]$



ϕ



f.



ψ

$f(\phi) \subseteq \psi$.

$f^{\#}(I)$

$I \to \gamma(I) = x \cdot \Rightarrow f(x) \Rightarrow f(\gamma(I))$

$\subseteq$

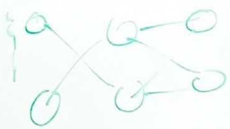
$f^{\#}(I) \Rightarrow \gamma(f^{\#}(I))$

$I = -2 \leq x \leq 4$.

$x = \{ p \mid -2 \leq p \leq 4 \}$. $-2, -1.99, \ldots +3.01 \ldots$

$f = \text{ReLU} = \text{ReLU}(-2), \text{ReLU}(-1.9) \ldots \text{ReLU}(3.01)\}$.

Φ



f.

Ψ

$f(\Phi) \subseteq \Psi$.

$f^{\#}(I)$

$I \to \gamma(I) = x \cdot \Rightarrow f(x) \to f(\gamma(I))$

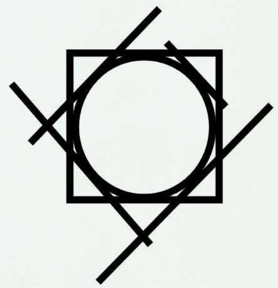
$\subseteq$

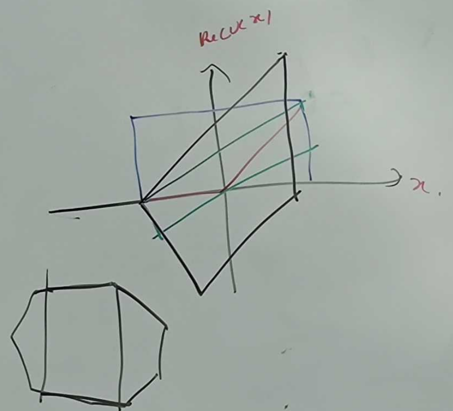
$f^{\#}(I) \Rightarrow \gamma(f^{\#}(I))$

$I = -2 \le x \le 4$.

$x = \{ p \mid -2 \le p \le 4 \}$, $-2, -1.99, \ldots, 3.01 \ldots$

$f = ReLU = ReLU(-2), ReLU(-1.9) \ldots ReLU(3.01)\}$.





Whenever you are exact $\Rightarrow$ Sound.

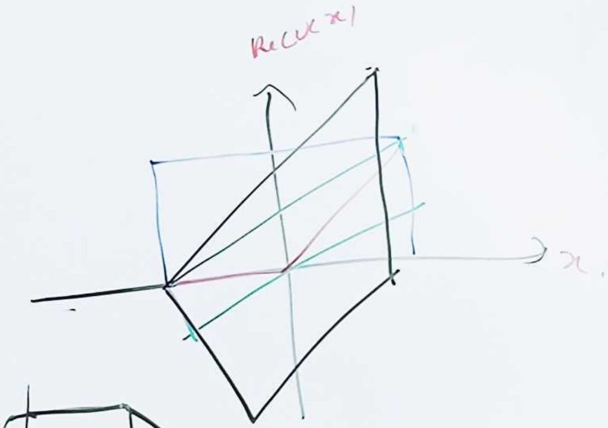
Optimal $\Rightarrow$ Sound.

exact $\Rightarrow$ Optimal.

optimality $\not\Rightarrow$ exactness.

Sound $\not\Rightarrow$ Exact

Sound $\not\Rightarrow$ optimal.



Whenever you are exact ⇒ Sound.
Optimal ⇒ Sound.

exact ⇒ Optimal.

optimality ⇏ exactness.

Sound ⇏ Exact

Sound ⇏ optimal.

$$\lambda * \left(\alpha_0 + \sum_{j=1}^{n} \alpha_i \varepsilon_i \right) \Rightarrow \lambda \alpha_0 + \sum_{j=1}^{n} \lambda \alpha_i \varepsilon_i$$

$[a, b)$

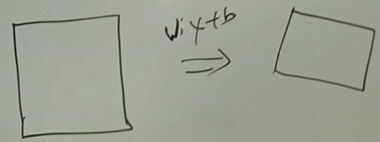
$$x = 1 - \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_1 = 1, \varepsilon_2 = 1 \quad 1$$

$[c, d]$ $\quad 1$

$$y = 1 + \varepsilon_1 - \varepsilon_2$$

$$x + y = (1+1) + (-1+1)\varepsilon_1 + (1-1)\varepsilon_2$$

$$= 2.$$



$$\lambda * \left(\alpha_0 + \sum_{i=1}^{n} \alpha_i \varepsilon_i \right) \Rightarrow \lambda \alpha_0 + \sum_{i=1}^{n} \lambda \alpha_i \varepsilon_i$$

(a, b)

$$x = 1 - \varepsilon_1 + \varepsilon_2 \Rightarrow \varepsilon_1 = 1, \varepsilon_2 = 1 \quad 1$$

$(c, d) \quad 1$

$$y = 1 + \varepsilon_1 - \varepsilon_2$$

$$x + y = (1+1) + (-1+1)\varepsilon_1 + (1-1)\varepsilon_2$$

$$= 2.$$